COMPUTER SIMULATION OF A PISTOL EXTERNAL BALLISTICS USING TWO VARIOUS AIR RESISTANCE LAWS

(on the example of the 7.62 mm TT pistol)

Vadim L. Khaikov
independent researcher, Krasnodar, Russian Federation,
e-mail: wadimhaikow@inbox.ru,
ORCID iD: 0000-0003-1433-3562

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Abstract:

Often by describing of a pistol or rifle cartridge, it is possible to see two ballistic coefficients that characterize the ballistic qualities with respect to various air resistance laws (ARL). How close are the obtained ballistic trajectories with varied specification to each other and what are the differences between them? How to evaluate ballistics if the ARLs are to be expressed in various mathematical forms?

In this paper the evaluation of pistol external ballistics trajectories for two air resistance laws (ARL) are given (ARL of the 1943 year and Siacci). All performed results relate to the TT pistol with 7.62 × 25 Tokarev cartridge. The paper presents also the answer to the question: how to calculate the ballistic trajectory if ARL is expressed as rational function, piecewise function or spline. For ARL of the 1943 year, a graphical interpretation of the function Cd (i, v) in the form of a surface and its main elements is shown.

It is present that due to the selection of ballistic coefficients; it is possible to obtain sufficiently similar in form ballistic trajectories.

A method of graphical comparison of ballistic trajectory parameters and mathematical tools for quantitative analysis of a shape of ballistic curves are represented. The difference between the two trajectories are proposed to estimate using of relative error in regard to selected ballistic parameter.

Computer simulation considered for ARLs of the 1943 year and Siacci for 7.62 × 25 Tokarev cartridge indicates that profiles of function instantaneous projectile velocity vs time of flight (TOF) had the greatest non-coincidence in relation to other ballistic parameters (e.g. horizontal range, height of the trajectory, etc.) The obtained maximum of relative error was 0.8%. Its magnitude localizes at the point of impact.

Key words: external ballistics, pistol TT, Tokarev cartridge, drag function, bullet trajectory, spline, Mathcad.

There are no dangerous weapons;
there are only dangerous men.
Robert A. Heinlein

Introduction

For one of the same projectiles (bullets), that have equal initial conditions (x_0, y_0, θ_0, v_0), but characterized different ARLs, it's possible to calculate so called "twins-trajectories". That are two or more trajectories having practically the identical form, but various ARLs description (for the same bullet) provide different values of ballistic coefficients *C*. The problem related with obtaining identical trajectories for various pre-selected ARLs is solved by iterative calculation of ballistic coefficients, but errors related with inequality of received trajectories are usually not reported.

Externall ballistics of bullets in Europe, in the United States and in countries of South America is based generally on the use of well-known *G* air drag models, however ARLs like the 1943 year and Siacci often used in Commonwealth of Independent States or in countries former members of the Warsaw Pact and in countries that had a military-technical cooperation with this defense treaty. One of the objectives of this article is to show how to carry out a ballistic simulating by using of the 1943 year and Siacci ARLs with various forms of their mathematical expression. The second task is to present the identity and the difference between the obtained during a computation process ballistical curves. For reduce a calculation time and for organization of a simulation process we will use the Mathcad 15 computer algebra system.

From the point of view of external ballistics, it is interesting to estimate ballistics one of the well-known pistols, for instance, of the 7.62 mm Tokarev-TT[1] pistol using two aforecited ARLs. It is known, that pistols based on the TT construction were produced in many countries and the 7.62×25 cartridge is widespread.

In the scientific article (Bogdanovich, 2012, p. 42) one can find «…one of the best pistols based on the 7.62 mm TT design was certainly the M57. This gun was constructed in Yugoslavia, at the «Zastava» plant and produced by Serbian «Zastava Arms» for export to various countries, including Europe and the USA». The arms plant «Crvena Zastava» (Kragujevac) began to produce the pistol-predecessor of the M57 namely the M54 in 1954 year and at the same time ammunition factory «Prvi Partizan» (Uzice) had launched a serial production of the 7.62×25 Tokarev

[1] 7.62 mm Tokarev self-loading pistol model TT 1930 (TT-30)/ TT 1933 (TT-33) abbreviated as TT (Tula-Tokarev)

cartridge. In addition, it should be said that the TT pistols and its upgrades were manufactured in PRC (Type 51 & Type 54), in Hungary (M48, Tokagypt 58 with cartridge 9x19mm Para), in Romania (TTC), in DPR Korea (Type 68) and in other countries.

Ballistical and technical data. In the Table 1 necessary technical specifications of TT-33, M54, M57 pistols, that important for evaluating their ballistics are demonstrated.

Table 1 – Technical specifications of the TT-33, M54, M57 pistols
Таблица 1 – Технические характеристики пистолетов TT-33, M54, M57
Табела 1 – Title

№/№	Specifications	Units	Model of pistol	
			TT-33, M54	M57
1	Chambering	mm	7.62 x 25 TT	7.62 x 25 TT
2	Fire modes	-	Semi-Auto, Single Action	Semi-Auto, Single Action
3	Bullet weight	g	5.49 - 5.52	5.49 - 5.52
4	Bullet length	mm	14[2]	-
5	Bore length	mm	116	116
6	Rifling length	mm	100	-
7	Number of grooves	-	4 RH	4 RH
8	Number of lands	-	4	4
9	Twist rate	mm per turn	240	240
		inch per turn	9.45	9.45
		clb [3] per turn	31.496	31.496
		° ' "	5°41'45"	5°41'45"
10	Initial velocity	mps	420	440
11	Bullet muzzle energy	Joule	485.54	532.88
12	Bullet spin rate	rps	1750	1750-1896
13	Effective firing range	m	50	50
14	Bullet flight range	m	800-1000	1640
15	Sight radius	mm	156	158
16	Max mean pressure	kg/cm^2	2234	-
17	Practical rate of fire	rpm	30	-
18	Precision (range: 50 m)	m	0.25	-

Sources: (Bogdanovich, 2012, p. 49) and author's estimations

Based on (http://popgun.ru, nd) curves of pressure and bullet velocity vs rifling length (time) for the 7.62 mm bullet of the TT-33 pistol in a logarithmic scale were built (Figure 1). The main advantage of the logarithmic scale is that it allows to us «to stretch the graph» in the direction

[2] for ordinary bullet P-type (cyrillic: пуля «П» - простая)
[3] clb – bullet caliber

to the origin (to the point «0 mm» by using argument bore length and to the point «0 seconds» by using argument time). The fragment *a* of the Figure 1 shows the change of the projectile velocity in the barrel; fragment *b* indicates the internal ballistic curve of mean pressure in the barrel.

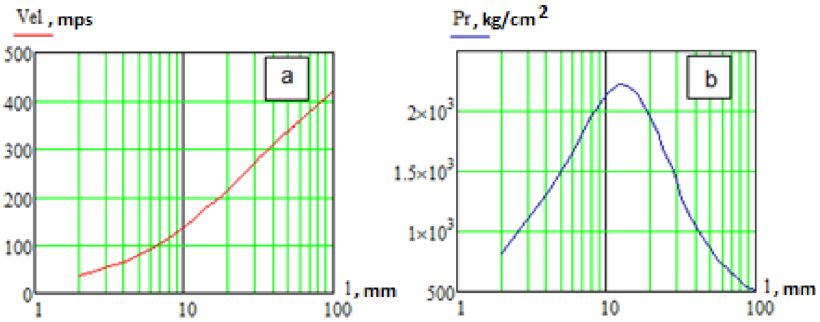

Figure 1 – Internal ballistic curves of the TT pistol (argument - bore length)
Рис. 1 – Внутрибаллистические кривые пистолета ТТ (аргумент – длина ствола пистолета)
Slika 1 – перевод на сербский.

The dependences of the bullet velocity and mean pressure as a function of time are also obtained (Figure 2). The graphs show that the bullet initial velocity is 420 mps, and the maximum of mean pressure is 2234 kg/cm^2. The duration of the intraballistic cycle for TT-33 pistol is approximately 2.5 milliseconds.

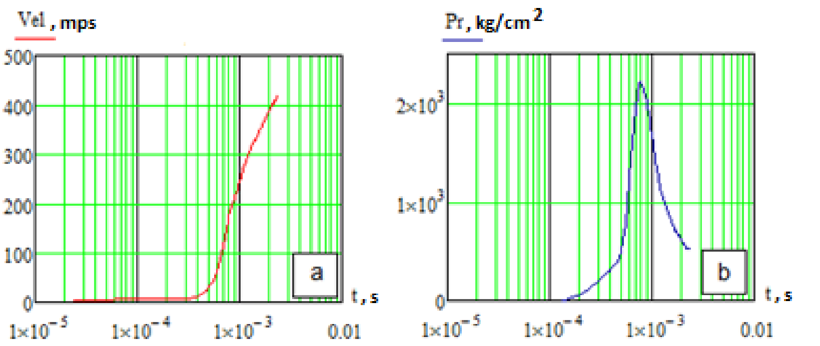

Figure 2 – Internal ballistic curves of the TT pistol (argument time): *a* – the projectile velocity in the barrel; *b* – internal ballistic curve of mean pressure in the barrel.
Рис. 2 – Внутрибаллистические кривые пистолета ТТ (аргумент – время): *а* – скорость снаряда в стволе; *b* – внутрибаллистическая кривая среднего давления в стволе.
Slika 2 – перевод на сербский.

The mathematical model. Longitudinal motion of a pistol's bullet in the Earth's atmosphere can be described by the system of ODEs with an independent argument TOF (*t*) (Cumin et al, 2009, p. 42). This type of mathematical expression is belonged to the type of Point-Mass Trajectory Model:

$$\begin{cases} \frac{dv}{dt} = -gsin(\theta) - \frac{\rho v^2}{2m} AC_d \\ \frac{d\theta}{dt} = -\frac{gcos(\theta)}{v} \\ \frac{dx}{dt} = vcos(\theta) \\ \frac{dy}{dt} = vsin(\theta) \end{cases} \quad (1)$$

here v – instantaneous bullet velocity, m/s;
t – time of flight, s;
g – acceleration of gravity at the point of departure, m/s^2;
θ – angle of velocity vector relative to the base of a trajectory, radian;
ρ – air density, kg/m^3;
m – mass of projectile, kg;
A – cross section of projectile, m^2;
C_d – a drag function, dimensionless;
x – the abscissa (horizontal range) of the trajectory, m;
y – the ordinate of the trajectory, m.

When using the standard ARLs and the coefficient *i*, from the first ODE equation of the system (1) takes the form:

$$\frac{dv}{dt} = -gsin(\theta) - \frac{\rho v^2}{2m} AiC_{d_{st}}$$

где i – coefficient taking into account a shape of (launched) bullet (so-called form coefficient[4]), dimensionless;
$C_{d_{st}}$ – the standard air drag function, dimensionless.

[4] In the book of Semikolenov, Bondarenko & Krasner «Principles of small unit weapons firing» (Semikolenov et al, 1971) on the p. 67 *i* is named as "the coefficient for the projectile shape".

Coefficients *i* and ARL models. According to collected sources *i* coefficients for the 7.62 mm pistol bullet and 9 mm bullets are gathered in the Table 2.

Table 2 – The values of the coefficient *i* for pistol bullets
Таблица 2 – Значение коэффициента *i* для пистолетных пуль
Tabela 2– перевод на сербский

Type of cartridge and bullet	Initial velocity V_0, mps	Bullet weight, g	ARL		Source
			1943	Siacci	
7.62×25 mm Tokarev	420	5.505	1.35	0.75	(Kirillov, 1963, p. 68)
9 x18 mm Makarov	315	6.1	-	0.98	(Vodorezov, 2017, p. 166)
9x19 mm Luger (FMJ)	376	7.4	1.526	0.77	(12)
9x19 mm Luger (HP)	308	9.4	1.509	0.755	(12)

A comparison of the values of the coefficients *i* for 7.62×25 mm Tokarev indicates, that its value for ARL of Siacci is 1.8 times less than *i* for ARL of the 1943 year.

The 1943 year drag model is often used to describe a ballistics of pistol bullets, for example, for 9 mm Para cartridge (Jankových, 2012, p. 29) or for 9 mm Luger (https://guns.ru/, nd).

The coefficient *i* can calculate by the following formula (Faraponov et al, 2017, p. 35)

$$i = \frac{m}{1000\, d^2}\, C \quad ,$$

where *i* – form coefficient, dimensionless;
m – mass of the projectile (bullet), kg;
d – caliber of the projectile (bullet), m;
C – ballistic coefficient of the projectile, m^2/kg.

«Although the coefficient *i* is usually regarded as a constant value, but, as can be seen from expression

$$i = C_d\left(\frac{v}{a}\right)\left(C_{d_{st}}\left(\frac{v}{a}\right)\right)^{-1} \quad ,$$

it, strictly speaking, depends on the instantaneous projectile velocity. Therefore, using a projectile (bullet) of the same shape in different ranges of velocity, we can get some discrepancies in the numerical values of the coefficient *i*. For the same reason, value of coefficient *i* for the same projectile and for the same initial velocity depends on the angle of departure (AOD). This is explained by the fact that changing of AOD gives a change in the velocity range along the trajectory» (Shapiro, 1946, p. 58).

For example, the relationship between ARL of the 1943 year and ARL of Siacci in the range of up to 5 *M* is shown in Figure 3 (Author, 2017, p. 83). The coefficient *i* as a function of the Mach number (*M*) is a complicated that it's not monotonous. Graph of the *i(M)* function has two local minimums and one local maximum (see right graphics window). However, *i(M)* can be characterized by some average value, which equal to half of the area under the *i(M)* function graph.

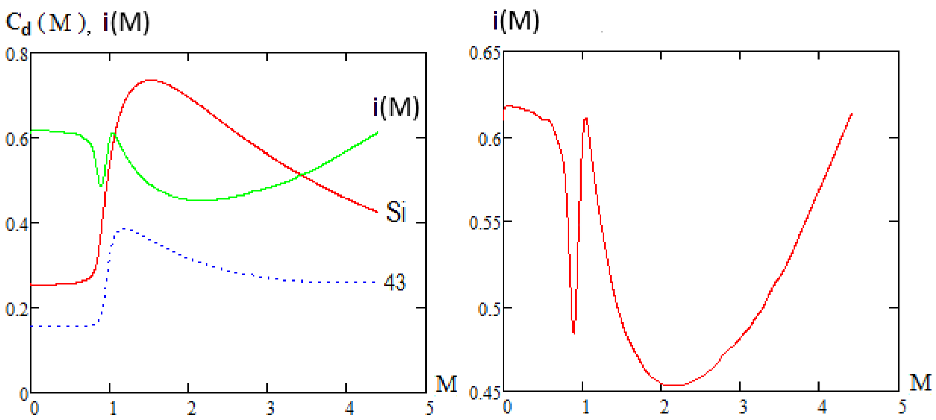

Figure 3 – The relationship between the two drag-functions
Рис. 3 – Отношение двух законов сопротивления воздуха
Slika 3 – перевод на сербский.

If *i(M)* and the coefficient *i* are considered as a some constant, then the function $C_d(i,v)$ can be represented as a surface (Figure 4). For *i=1*, we have a standard function $C_{d_{st}}\left(\frac{v}{a}\right)$, which is a section of the surface (orange line). For *i≠1*, the individual function C_d, such as a purple line (*i>1*).

Using the value of the coefficient *i*, the caliber of the bullet, its mass, it is possible to calculate the ballistic coefficient *C* (Germershausen, 1982, p. 159)

$$C = 1000 \cdot i \cdot d^2 \cdot q^{-1} \quad .$$

In this case we obtain the function $C_d(C, v)$.

However, there is an alternative formula for calculating the ballistic coefficient (BC)

$$BC = \frac{m}{d^2 \cdot i} \quad .$$

In order to avoid misunderstanding it is necessary to indicate a type of a calculation formula for ballistic coefficient determination.

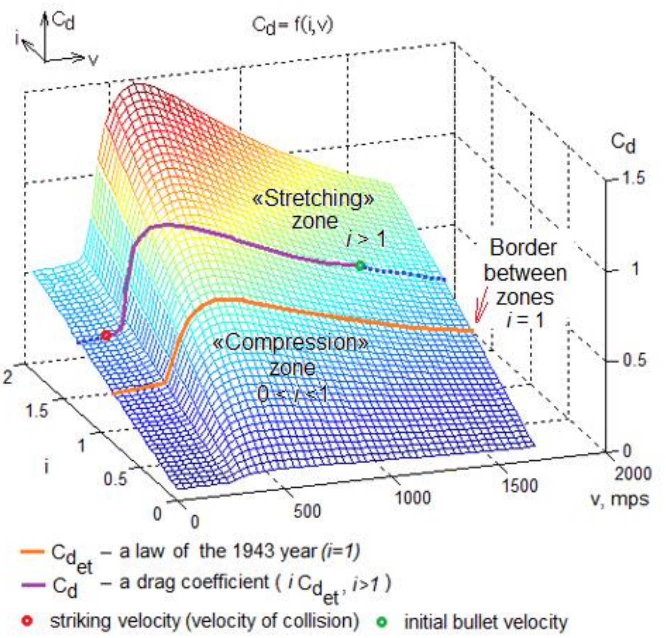

Figure 4 – The function *Cd(i,v)* in the form of a surface
Рис. 4 – Функция *Cd(i,v)* в форме поверхности
Slika 4 – перевод на сербский.

So, if *i(v) = const*, then the surface (Figure 4) shows all possible individual drag-functions which depending on *i*. When the coefficient *i* is multiplied by $C_{d_{st}}\left(\frac{v}{a}\right)$, a linear transformation of the function takes place: for *i > 1*, the graph is stretched from the abscissa axis *i* times; for *0 < i <1* this is the compression of the graph to the *x*-axis by 1/*i* times.

Therefore, the standard function $C_{d_{st}}\left(\frac{v}{a}\right)$ is the boundary between the «compression» and «stretching» zones of the *Cd(i,v)* surface. Since any ballistic trajectory has the initial and striking velocity of the projectile, due to the surfaces $C_d(i,v)$ or $C_d(C,v)$ we can show the range of the coefficient C_d, that necessary for flight path calculations.

Different forms of ARL expressions. ARLs or $C_{d_{et}}$(v) function can be described as: an analytical function; a piecewise function; a spline function. The spline function can be regarded as a special kind of piecewise function, functions-pieces of which are cubic polynomials.

Analytical forms for ARL of the 1943 year. In view of the fact that the summit of the bullet trajectory in air for pistol external ballistics is not large value the speed of sound can be considered as a constant value. That is in formula

$$M = v/a,$$

where v – instantaneous bullet velocity; a – local speed of sound (constant).

ARL of the 1943 year is a tabular function that can be found in (Konovalov et al, 1979, p. 191) and approximated using a rational function:

$$C_{d_{43_{RF}}}\left(\frac{v}{a}\right) = \frac{P\left(\frac{v}{a}\right)}{Q\left(\frac{v}{a}\right)} \quad .$$

Investigating possible approximation forms for the ARL of the 1943 year, it proposed the following rational function (Author, 2017, p. 85):

$$C_{d_{43_{RF}}}\left(\frac{v}{a}\right) = \frac{1.378212 - 7.1379605\left(\frac{v}{a}\right) + 15.498681\left(\frac{v}{a}\right)^2}{8.7777403 - 45.498974\left(\frac{v}{a}\right) + 99.290858\left(\frac{v}{a}\right)^2} \cdot\cdot$$

$$\cdot\cdot\frac{-17.778376\left(\frac{v}{a}\right)^3 + 10.605229\left(\frac{v}{a}\right)^4 - 1.7807148\left(\frac{v}{a}\right)^5}{-17.778376\left(\frac{v}{a}\right)^3 + 74.91108\left(\frac{v}{a}\right)^4 - 21.331814\left(\frac{v}{a}\right)^5} \cdot\cdot$$

$$\cdot\cdot\frac{-1.6876336\left(\frac{v}{a}\right)^6 + 1.164362\left(\frac{v}{a}\right)^7 - 0.2873904\left(\frac{v}{a}\right)^8}{-3.0222138\left(\frac{v}{a}\right)^6 + 4.0786158\left(\frac{v}{a}\right)^7 - 1.0962723\left(\frac{v}{a}\right)^8} \cdot\cdot \qquad (2)$$

$$\cdot\cdot\frac{+0.025985844\left(\frac{v}{a}\right)^9}{+0.1012291\left(\frac{v}{a}\right)^9} .$$

Another form of mathematical expressing for ARL of the 1943 year set of rational and exponential functions (R&EF) (Kozlitin et al, 2016, p. 29):

$$C_{d_{43R\&EF}}\left(\frac{v}{a}\right)=\frac{P\left(\frac{v}{a}\right)}{Q\left(\frac{v}{a}\right)}+\frac{b_0}{1+exp\left(b_1\left(\frac{v}{a}\right)+b_2\right)}+d_0 \tag{3}$$

It should be noted that the rational function (i.e. first summand) uses only even powers (from 0 to 12, namely 0, 2, 4,..10, 12). R&EF is expressed by the following formula

$$C_{d_{43R\&EF}}\left(\frac{v}{a}\right)=\frac{-1.9382+4.2980\left(\frac{v}{a}\right)^2+0.3207\left(\frac{v}{a}\right)^4}{296.9213-853.9492\left(\frac{v}{a}\right)^2+985.5873\left(\frac{v}{a}\right)^4}\cdot\cdot$$

$$\cdot\cdot\frac{-9.4610\left(\frac{v}{a}\right)^6+8.9342\left(\frac{v}{a}\right)^8-0.9476\left(\frac{v}{a}\right)^{10}}{-580.8643\left(\frac{v}{a}\right)^6+178.6690\left(\frac{v}{a}\right)^8-15.4071\left(\frac{v}{a}\right)^{10}}\cdot\cdot \tag{4}$$

$$\cdot\cdot\frac{+0.0525\left(\frac{v}{a}\right)^{12}}{+1.0000\left(\frac{v}{a}\right)^{12}}+\frac{0.0531}{1+exp\left(-90.5063\left(\frac{v}{a}\right)+85.5194\right)}+0.1639$$

Research conducted in (Author, 2017, p. 88) showed that matrixes **P**, **Q**, **B** and **D** in the formula (3) may have a different coefficients. For example, matrices $\mathbf{P_1}$, $\mathbf{Q_1}$, $\mathbf{B_1}$ and $\mathbf{D_1}$ with alternative coefficients are presented below:

$$P_1:=\begin{pmatrix}10.189924313\\-32.2497749054\\42.0499139169\\-28.8388279297\\9.989953385\\-0.6976279168\\0.0403773785\end{pmatrix}\qquad Q_1:=\begin{pmatrix}-0.9050248435\\2.8653174742\\-3.7411757325\\2.5742663163\\-0.8961295841\\0.0627439736\\-0.0036275618\end{pmatrix}$$

$$B_1 := \begin{pmatrix} 0.06274397 \\ 16.399062 \\ 57.358636 \end{pmatrix} \qquad D_1 := \begin{pmatrix} 11.416713 \end{pmatrix}$$

ARL of the 1943 year can be expressed as a piecewise function (5) (Author, 2017, p. 80) consisting of 9 unequal intervals. This function is a modification of the formula from (Konovalov et al, 1979, p. 84) with a large number of intervals-pieces:

$$C_{d_{43PWF}}\left(\frac{v}{a}\right) = \\ = \left(\begin{array}{cc} 0.157 & 0.1 < \left(\frac{v}{a}\right) \le 0.73 \\ 0.033\left(\frac{v}{a}\right) + 0.133 & 0.73 < \left(\frac{v}{a}\right) < 0.82 \\ 3.9\left(\frac{v}{a}\right)^2 - 6.4194\left(\frac{v}{a}\right) + 2.8025831 & 0.82 \le \left(\frac{v}{a}\right) < 0.91 \\ 1.5\left(\frac{v}{a}\right) - 1.176 & 0.91 \le \left(\frac{v}{a}\right) \le 1.00 \\ -1.6\left(\frac{v}{a}\right)^2 + 3.7632\left(\frac{v}{a}\right) - 1.8287616 & 1.00 < \left(\frac{v}{a}\right) \le 1.18 \\ 0.384 sin\left(1.85\left(\frac{v}{a}\right)^{-1}\right) & 1.18 < \left(\frac{v}{a}\right) < 1.62 \\ 0.29\left(\frac{v}{a}\right)^{-1} + 0.172 & 1.62 \le \left(\frac{v}{a}\right) < 3.06 \\ -0.011\left(\frac{v}{a}\right) + 0.301 & 3.06 \le \left(\frac{v}{a}\right) \le 3.53 \\ 0.259 & 3.53 \le \left(\frac{v}{a}\right) \le 4.0 \end{array} \right) \tag{5}$$

Sometimes the last interval of (5) is extended to 5 *M*. The subscript «*PWF*» denotes a piecewise function.

Analytical forms for ARL of Siacci. *F*-curve for Siacci function is written as follows (Mori, 2013, p. 41)

$$F_{si}(v) = 0{,}2002v - 48{,}05\sqrt{(0{,}1648v - 47{,}95)^2 + 9{,}6} \\ + \frac{0{,}0442v(v - 300)}{371 + \left(\frac{v}{200}\right)^{10}} \tag{6}$$

Due to division by $4.74 \cdot 10^{-4} v^2$ we can transform the *F*-curve into the C_d-type function (Shapiro, 1946, p. 37)

$$C_{d_{si}}(v) = \frac{F_{si}(v)}{4.74 \cdot 10^{-4} v^2} \tag{7}$$

Siacci ARL as tabular function can be found in (Konovalov et al, 1979, p. 191).

The technology of using spline functions is demonstrated in (Author, 2018) and in this article will not be considered in detail.

Forms of mathematical notation for ARLs of the 1943 year and Siacci reviewed in present research paper are collected in Table 3.

Table 3 – Forms of mathematical notation for ARLs of the 1943 year and Siacci
Таблица 3 – Формы математических обозначений законов сопротивления воздуха 1943 года и Сиаччи
Tabela 3– перевод на сербский

ARL	Analytical function		Table-function
	Classical analytical type	Piecewise type	
1943	formulas 2, 3, 4	formula 5	* [5]
Siacci	formulas 6, 7	-	*

Mathcad programming code. Commented Mathcad-code is presented below. Make determination of the characteristics for a pistol's bullet: caliber (0.00762 *m* = 7.62 *mm*), weight (0.0055 *kg*) and a value of the *i* coefficient (*i_43*) (according to the the chosen law):

d:=0.00762　　　*q:=0.0055*　　　*i_43:=1.35*

An angle of departure (in radians) is calculated as a set of angular degrees, minutes and seconds:

Gradus:=0　　　*Min:=10*　　　*Sec:=0*

$$\theta_0 := \frac{\pi}{180}\left(Gradus + \frac{Min}{60} + \frac{Sec}{3600}\right) = 2.909 \cdot 10^{-3}$$

At the point of departure it is determined the value of the acceleration coefficient of gravity as 9.18 m/s^2. Further it is necessary to determine the

5 * - source corresponds (Konovalov et al, 1979, p. 191).

time interval of integration: its boundaries and the total number of integration points:

t_{beg}:= *0* t_{end}:= *1.1* n_{points}:= 1000

The initial conditions (for formula (1)) are determinate as a matrix-column *y*, which will contain their known numerical values:

$$y := \begin{pmatrix} y_0 = v(0) \\ y_1 = \theta(0) \\ y_2 = x(0) \\ y_3 = y(0) \end{pmatrix}.$$

In view of the fact that the initial velocity of the 7.62 mm TT bullet is 420 m/s the matrix-column *y* will look like:

$$y := \begin{pmatrix} 420 \\ 2.051 \times 10^{-3} \\ 0 \\ 0 \end{pmatrix}.$$

The matrix *D(t, y)* has the form

$$\mathrm{D}\ (\mathrm{t,y}) := \begin{bmatrix} -\mathrm{g}\cdot\sin(\mathrm{y2}_1) - \dfrac{\rho\cdot(\mathrm{y}_0)^2}{2\cdot\mathrm{m}}\cdot\mathrm{A}\cdot\mathrm{i\_43}\cdot\mathrm{Cd}\left(\dfrac{\mathrm{y}_0}{\mathrm{a}}\right) \\ \dfrac{-\mathrm{g}\cdot(\cos(\mathrm{y}_1))^2}{\mathrm{y}_0} \\ \mathrm{y}_0\cdot\cos(\mathrm{y}_1) \\ \mathrm{y}_0\cdot\sin(\mathrm{y}_1) \end{bmatrix}. \qquad (8)$$

In connection with the fact that C_d in formulas (2), (3) or other is a long and cumbersome expression, it is given in (8) only as a short notation.

If a real solution in the Mathcad system is implemented, then C_d must be replaced by the complete formula (see the appendix to this article). In this formula, the sign of *v* (velocity) is replaced by y_0. *i_43* – form coefficient of the 1943 year in Mathcad program.

For example, the matrix-column *D(t, y)* will have the form (for C_d only three initial terms are given; degrees of $\frac{y_0}{a}$ are from 0 to 2). A Mathcad script for *D(t, y)* is given below.

$$D(t,y) := \begin{bmatrix} -g\cdot\sin(y_1) - \dfrac{\rho\cdot(y_0)^2}{2\cdot m}\cdot A\cdot i\_43\,\dfrac{1.378212 - 7.1379605\left(\dfrac{y_0}{a}\right) + 15.498681\left(\dfrac{y_0}{a}\right)^2 + \ldots}{8.7777403 - 45.498974\left(\dfrac{y_0}{a}\right) + 99.290858\left(\dfrac{y_0}{a}\right)^2 + \ldots} \\ \dfrac{-g\cdot(\cos(y_1))^2}{y_0} \\ y_0\cdot\cos(y_1) \\ y_0\cdot\sin(y_1) \end{bmatrix}$$

The complete Mathcad script of $C_{d_{43_{RF}}}\left(\frac{y_0}{a}\right)$ and $C_{d_{43_{R\&EF}}}\left(\frac{y_0}{a}\right)$ expressions is given in the appendix to this article.

In order to use the piecewise function (5) in computer calculations, we transform it to the form, that used in the Mathcad:

$$D(t,y) := \begin{bmatrix} -g\cdot\sin(y_1) - \dfrac{\rho(y_0)^2}{2\cdot m}\cdot A\cdot i\_43 \left| \begin{array}{l} 0.157 \text{ if } \left(0.1 \le \dfrac{y_0}{a} \le 0.73\right) \\ \left(0.033\,\dfrac{y_0}{a} + 0.133\right) \text{ if } \left(0.73 < \dfrac{y_0}{a} \le 0.82\right) \\ \left[3.9\left(\dfrac{y_0}{a}\right)^2 - 6.4194\,\dfrac{y_0}{a} + 2.8025831\right] \text{ if } \left(0.82 < \dfrac{y_0}{a} \le 0.91\right) \\ \left(1.5\,\dfrac{y_0}{a} - 1.176\right) \text{ if } \left(0.91 < \dfrac{y_0}{a} \le 1.0\right) \\ \left[-1.6\left(\dfrac{y_0}{a}\right)^2 + 3.7632\,\dfrac{y_0}{a} - 1.8287616\right] \text{ if } \left(1.0 < \dfrac{y_0}{a} \le 1.18\right) \\ \left(0.384\sin\left(\dfrac{1.85}{\dfrac{y_0}{a}}\right)\right) \text{ if } \left(1.18 < \dfrac{y_0}{a} \le 1.62\right) \\ \left(0.29\,\dfrac{1}{\dfrac{y_0}{a}} + 0.172\right) \text{ if } \left(1.62 < \dfrac{y_0}{a} \le 3.06\right) \end{array} \right. \end{bmatrix}$$

Continuation of formula for previous page

$$\begin{bmatrix} \left| \begin{array}{l} \left(-0.011\frac{y_0}{a}+0.301\right) \text{ if } \left(3.06<\frac{y_0}{a}\le 3.53\right) \\ 0.259 \text{ if } \left(3.53<\frac{y_0}{a}\le 5.0\right) \end{array} \right. \\ \frac{-g\cdot(\cos(y_1))^2}{y_0} \\ y_0\cdot\cos(y_1) \\ y_0\cdot\sin(y_1) \end{bmatrix}$$

The matrix-column *D(t, y)* (9) is the right-hand parts of the system of ODEs (1). Its includes variables with the following notation: the instantaneous projectile velocity $v - y_0$, angle of inclination of the tangent $\theta - y_1$, the abscissa of the trajectory $x - y_2$; ordinate of the trajectory $y - y_3$:

$$D(t,y) := \begin{bmatrix} -g\cdot\sin(y2_1) - \frac{\rho\cdot(y_0)^2}{2\cdot m}\cdot A\cdot i\_43\cdot Cd\left(\frac{y_0}{a}\right) \\ \frac{-g\cdot(\cos(y_1))^2}{y_0} \\ y_0\cdot\cos(y_1) \\ y_0\cdot\sin(y_1) \end{bmatrix}. \quad (9)$$

Below we give an example of the matrix *D(t, y)* for the description of ARL and system of the ODEs (1) by using a cubic spline. Import of the table-function of ARL data is carried out from an external file. Its can be a text «.txt» file or an Microsoft Excel «.xls» file.

$$D(t,y) := \begin{bmatrix} -g\cdot\sin(y_1) - \frac{\rho\cdot(y_0)^2}{2\cdot m}\cdot A\cdot i\_43\cdot \text{interp}(\text{cspline}(vel, Cd), vel, Cd, y_0) \\ \frac{-g\cdot(\cos(y_1))^2}{y_0} \\ y_0\cdot\cos(y_1) \\ y_0\cdot\sin(y_1) \end{bmatrix}$$

Alternatively, the data for the ARL may not be imported from file, but be part of the *D(t, y)* matrix. In this case, data are written in the form of row-matrices. Separated for velocities and separately for C_d data. Next, as in the previous example, we use cubic spline interpolation.

$$D(t,y) := \begin{bmatrix} -g\cdot\sin(y_1) - \dfrac{\rho\cdot(y_0)^2}{2\cdot m}\cdot A\cdot i\_43 \cdot \left[\begin{array}{|l} vel \leftarrow (0\ \ 200\ \ 400\ \ \blacksquare\blacksquare)^T \\ Cd \leftarrow (7.105e\text{-}15\ \ 4.921\ \ 51.533\ \ \blacksquare\blacksquare)^T \\ interp(cspline(vel, Cd), vel, Cd, y_0) \end{array}\right] \\ \dfrac{-g\cdot(\cos(y_1))^2}{y_0} \\ y_0\cdot\cos(y_1) \\ y_0\cdot\sin(y_1) \end{bmatrix}$$

Four black rectangles in two lines (*vel* and C_d) symbolize the remaining elements of the matrix-lines *vel* (tabulated instantaneous velocity) and C_d (drag coefficient). The sign of T denotes a matrix transposition.

The following Mathcad command-line is showed a using of the solver-function *rkfixed* for numerical solution of (1) (Kir'yanov D.V., 2012, p. 259):

$$\text{Num\_Result} := \text{rkfixed}(y,\ t_{beg},\ t_{end},\ n_{point},\ D)\ .$$

Solver-function *rkfixed* is implemented the non-stiff fourth order Runge-Kutta numerical method with a fixed step.

Calculations. Calculations were performed for ARL of 1943 year for 4 C_d-types (RF, R&EF, piecewise, spline-function) and 2 types of the Ciacci ARL (analytical and spline-function).

The variable *Num_Result* is the matrix that containing the results of the numerical solution of the (1). In this case, the matrix has a dimension of 5 × 1001 elements and contains 5005 numbers. Five columns of the *Num_Result* matrix are: independent argument TOF (*t*); and elements of the matrix *y* (or *D*) namely v, θ, x, y. 1001 rows are the sum of 1 (initial condition) and n_{points}. The first row of matrix *Num_Result* includes $t(0)$, $v(0)$, $\theta(0)$, $x(0)$, $y(0)$. The first column of matrix *Num_Result* contains 1001 discrete TOF values: from $t_{beg} = t(0)$ to t_{end} . The 5-by-1001 matrix from the second to fifth columns (in each of them) has 1001 values of the quantities v, θ, x, y respectively. This means that we have 1001 values of instantaneous velocity, 1001 values of θ, 1001 values of x and so on.

Analysis of results. Previously it was shown, that for obtaining two trajectories characterized by different ARLs same horizontal ranges it is necessary to make selection of ballistic coefficients. This procedure allows to obtain sufficiently close forms of both ballistic trajectories. However, due to the fact, that the bullet movement for each flight paths is determined by the intrinsic ARL, that the bullet retardation process will not coincide with "twins-trajectories". For comparison of dependencies between elements of the ODEs (1), the method developed in (Author, 2018, p. 00) will be used below. The solution of the system (1) is represented as a five-dimensional space. Each element of this 5D-space is a function between the variables (*x, y, θ, v*) and argument (t), which are obtained as a result of the numerical solution (1). The angle *θ* is calculated in angular minutes (or minute of angle (MOA)). Since the solution of the system (1) depends on the ballistic coefficient, it becomes possible to compare the same-named dependencies (x_1, y_1, θ_1, v_1, t_1), (x_2, y_2, θ_2, v_2, t_2) obtained for different values of coefficients c_1 and c_2. The entire set of relations between variables and independent argument of (1) is presented in Table 4. The order of values (*x*, *y*, *θ*, *v*, *t*) location in the *5-by-1001* matrix *Num_Result* and in the Table 4 is different.

Table 4 – The structure of the relations for ODEs parameters (1) for two ARLs
Таблица 4 – Структура соотношений параметров системы дифференциальных уравнений (1) при различных законов сопротивления
Tabela 4– перевод на сербский

		X_1, X_2	Y_1, Y_2	θ_1, θ_2	V_1, V_2	T_1, T_2
		1	2	3	4	5
X_1, X_2	1	X_1 vs X_1 X_2 vs X_2	X_1 vs Y_1 X_2 vs Y_2	X_1 vs θ_1 X_2 vs θ_2	X_1 vs V1 X_2 vs V2	X1 vs T1 X2 vs T2
Y_1, Y_2	2	Y_1 vs X_1 Y_2 vs X_2	Y_1 vs Y_1 Y_2 vs Y_2	Y_1 vs θ_1 Y_2 vs θ_2	Y1 vs V1 Y2 vs V2	Y1 vs T1 Y2 vs T2
θ_1, θ_2	3	θ_1 vs X_1 θ_2 vs X_2	θ_1 vs Y_1 θ_2 vs Y_2	θ_1 vs θ_1 θ_2 vs θ_2	θ1 vs V1 θ2 vs V2	θ1 vs T1 θ2 vs T2
V_1, V_2	4	V_1 vs X_1 V_2 vs X_2	V_1vs Y_1 V_2vs Y_2	V_1 vs θ_1 V_2 vs θ_2	V1 vs V1 V2 vs V2	V1 vs T1 V2 vs T2
T_1, T_2	5	T_1 vs X_1 T_2 vs X_2	T_1 vs Y_1 T_2 vs Y_2	T_1 vs θ_1 T_2 vs θ_2	T1 vs V1 T2 vs V2	T1 vs T1 T2 vs T2

- relationship between the variables of ODEs (1);
- relationship between variables (1) and argument (t);
- diagonal cells.

The graphs lying inside the green background are functions between variables of the ODE (1) (x, y, θ, v). The graphs located inside the blue background associate the variables with the argument TOF (t). Diagonal cells - graphs placed on a light yellow background of graphics window show functions depending from itself, for example, «y is a function of y» and so on. A small red square on each of the 25 graphs shows the starting point. If we plot a horizontal and vertical line through starting points of any graph (see Figure 5), they will connect starting points of the graphs along the vertical row and horizontal line.

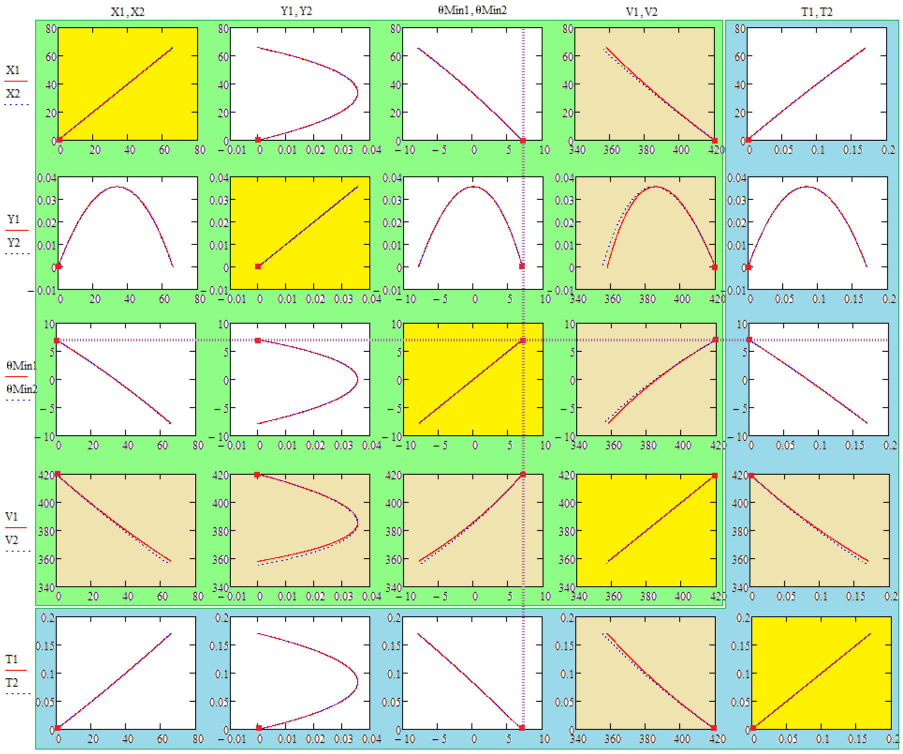

Figure 5 – The relationship between the ballistic parameters[6] ODEs (1) for two different ARLs
Рис. 5 Соотношение между баллистическими параметрами системы (1) при двух разных законах сопротивления воздуха
Slika 5 – перевод на сербский.

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[6] each of the 25 windows of Figure 5 contains 2 graphics, characterizing the ballistic parameters in relation to the 1943 ARL and the Siacci ARL.

Error analysis. Determination of the magnitude of the relative error (MRE) for pistol ballistics using two different ARL is an important element of assessments. To do this, we find the MRE of the horizontal range, the height of the trajectory, the angle of inclination of the velocity vector and the instantaneous velocity as functions of TOF. Relative error is expressed in percent.

Evaluation of MRE for the horizontal range is

$$\delta_X(t) = 100\frac{|x(t)_{43} - x(t)_{Si}|}{x(t)_{43}}.$$

MRE for the height of the trajectory is

$$\delta_X(t) = 100\frac{|x(t)_{43} - x(t)_{Si}|}{x(t)_{43}}.$$

Evaluation of MRE for instantaneous velocity of the projectile is

$$\delta_v(t) = 100\frac{|v(t)_{43} - v(t)_{Si}|}{v(t)_{43}}.$$

MRE for the angle of velocity vector relative to the base of a trajectory

$$\delta_\theta(t) = 100\frac{|\theta(t)_{43} - \theta(t)_{Si}|}{\theta(t)_{43}}.$$

The subscript «43» denotes that the calculations characterize the ARL of 1943 years, and «Si» for Siacci.

The results of the calculations are shown on Figure 6. The MRE is found for a TOF interval of 0-1.1 *s*. The time of 1.1 seconds corresponds to the time of impact.

Figure 6*a* gives the function MRE of the horizontal range vs TOF $\delta_X(t)$ and the instantaneous velocity vs TOF $\delta_v(t)$. The results of the calculations and comparisons show that $\delta_X(t)$ and $\delta_v(t)$ are a TOF-increasing functions. The maximum MRE for $\delta_X(t)$ is 0.3% for TOF 1.1 seconds. A function characterizing the MRE for the instantaneous velocity $\delta_v(t)$ has a similar character (figure 6*a*). The maximum value of this function is 0.8% (for same TOF point).

The result of dividing function $\delta_v(t)$ by function $\delta_X(t)$ is shown on Figure 3. Thus, MRE for the instantaneous velocity is approximately 2.8-3 times larger than MRE for the horizontal range.

In contrast to the functions mentioned above, the functions $\delta_\theta(t)$, $\delta_Y(t)$ don't have an increasing character, and moreover they have discontinuities (figures 6*c*, 6*d*). The discontinuity for the function $\delta_\theta(t)$ corresponds to the vertex of the trajectory. At this point, the angle θ is zero (division by zero). The discontinuity for the function $\delta_Y(t)$ corresponds to the point of impact. At this point, the height of the trajectory *y* is zero too and we have division by zero.

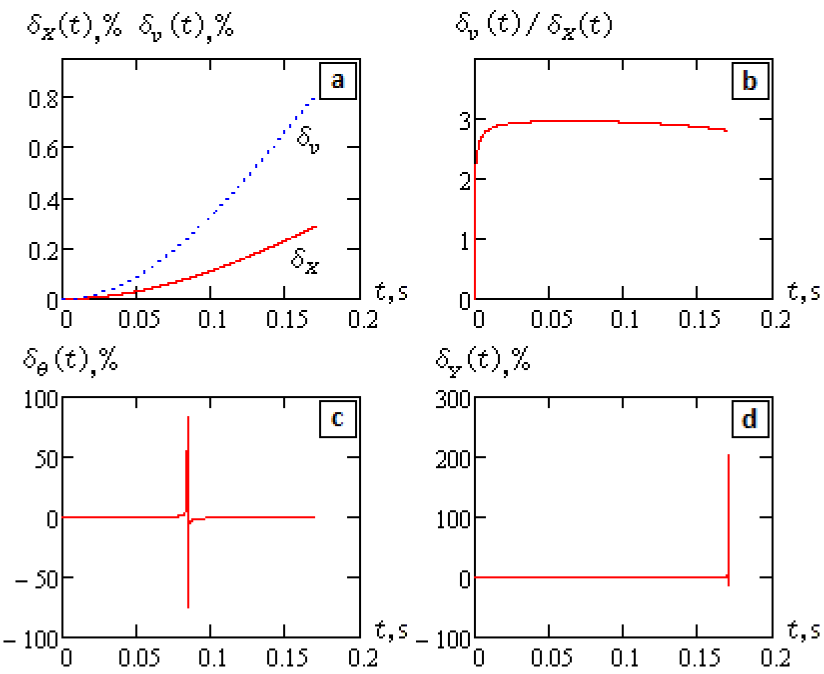

Figure 6 – Results of the error analysis: $\delta_X(t)$, $\delta_v(t)$, $\delta_\theta(t)$, $\delta_Y(t)$
Рис. 6 Результаты анализа погрешностей: функции $\delta_X(t)$, $\delta_v(t)$, $\delta_\theta(t)$, $\delta_Y(t)$
Slika 6 – перевод на сербский.

In contrast to the functions mentioned above, the functions $\delta_\theta(t)$ and $\delta_Y(t)$ don't have an increasing character as functions $\delta_X(t)$, $\delta_v(t)$ and moreover they have discontinuities. The discontinuity for the function $\theta(t)$ corresponds to the vertex of the trajectory (*t=0.084* s.). At this point, the $\theta(t)$ = 0 (division by zero). The discontinuity for the function $\delta_Y(t)$ corresponds to the point of collision ($t=1.1$ s). At this point, the height of the trajectory is zero (division by zero).

Summary and conclusions

Evaluations and external ballistics trajectories of TT pistol with cartridge 7.62×25 Tokarev are given for two ARLs (the 1943 year and Siacci). The characteristic of the internal-ballistic period for TT is shown. Feature of calculation is a using of various forms of ARL mathematical notation: classical analytical formulas, piecewise formula and function-tables form.

For ARL of the 1943 year a graphical interpretation of $C_d(i, v)$ function in the form of a surface and its main elements is visualized. Depending on the value of form coefficient i, it's demonstrated how the standard drag-function $C_d(v)$ is transformed.

A method of graphical comparison of ballistic trajectory parameters is represented. This comparison takes place in a matrix with dimension 5×5.

It is shown that due to the selection of ballistic coefficients, it is possible to obtain sufficiently close "twins-trajectories". However, in connection with the fact that the movement of the bullet in each of them is determined by different ARLs, then the slowing down of the bullet on each of them will have its own independent nature and therefore do not coincide with the "twins-trajectories".

Computer simulation considered for ARLs of the 1943 year and Siacci for 7.62 × 25 Tokarev cartridge indicates that profiles of function instantaneous projectile velocity vs time of flight had the greatest non-coincidence in relation to other ballistic parameters (e.g. horizontal range, height of the trajectory, etc.) The obtained maximum of relative error was 0.8%. Its magnitude localizes at the point of impact.

The results of simulation are shown that MRE for the instantaneous velocity is approximately 2.8-3 times larger than MRE for the horizontal range.

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Appendix

Mathcad-form of mathematical expressing for ARL of the 1943 year with 20 approximatively coefficients (Formula 2 – RF rational function)

$$C_{d_{43RF}}\left(\frac{y_0}{a}\right) = \frac{1.378212 - 7.1379605\left(\frac{y_0}{a}\right) + 15.498681\left(\frac{y_0}{a}\right)^2}{8.7777403 - 45.498974\left(\frac{y_0}{a}\right) + 99.290858\left(\frac{y_0}{a}\right)^2} \cdot\cdot$$

$$\cdot\cdot\frac{-17.778376\left(\frac{y_0}{a}\right)^3 + 10.605229\left(\frac{y_0}{a}\right)^4 - 1.7807148\left(\frac{y_0}{a}\right)^5}{-17.778376\left(\frac{y_0}{a}\right)^3 + 74.91108\left(\frac{y_0}{a}\right)^4 - 21.331814\left(\frac{y_0}{a}\right)^5} \cdot\cdot$$

$$\cdot\cdot\frac{-1.6876336\left(\frac{y_0}{a}\right)^6 + 1.164362\left(\frac{y_0}{a}\right)^7 - 0.2873904\left(\frac{y_0}{a}\right)^8}{-3.0222138\left(\frac{y_0}{a}\right)^6 + 4.0786158\left(\frac{y_0}{a}\right)^7 - 1.0962723\left(\frac{y_0}{a}\right)^8} \cdot\cdot$$

$$\cdot\cdot\frac{+0.025985844\left(\frac{y_0}{a}\right)^9}{+0.1012291\left(\frac{y_0}{a}\right)^9}.$$

Mathcad-form of mathematical expression for ARL of the 1943 year with 18 approximatively coefficients (Formula 4 – R&EF rational and exponential functions).

$$C_{d_{43R\&EF}}\left(\frac{y_0}{a}\right) = \frac{-1.9382 + 4.2980\left(\frac{y_0}{a}\right)^2 + 0.3207\left(\frac{y_0}{a}\right)^4}{296.9213 - 853.9492\left(\frac{y_0}{a}\right)^2 + 985.5873\left(\frac{y_0}{a}\right)^4} \cdot\cdot$$

$$..\frac{-9.4610\left(\frac{y_0}{a}\right)^6+8.9342\left(\frac{y_0}{a}\right)^8-0.9476\left(\frac{y_0}{a}\right)^{10}}{-580.8643\left(\frac{y_0}{a}\right)^6+178.6690\left(\frac{y_0}{a}\right)^8-15.4071\left(\frac{y_0}{a}\right)^{10}}..$$

$$..\frac{+0.0525\left(\frac{y_0}{a}\right)^{12}}{+1.0000\left(\frac{y_0}{a}\right)^{12}}+\frac{0.0531}{1+exp\left(-90.5063\left(\frac{y_0}{a}\right)+85.5194\right)}+0.1639$$

In the matrix (8) and in the matrix of the initial conditions (**Y**), y_0 denotes the instantaneous velocity of the gun projectile.

КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ВНЕШНЕЙ БАЛЛИСТИКИ ПИСТОЛЕТА С ИСПОЛЬЗОВАНИЕМ ДВУХ РАЗЛИЧНЫХ ЗАКОНОВ СОПРОТИВЛЕНИЯ ВОЗДУХА
(на примере пистолета 7.62 мм ТТ)

Вадим Л. Хайков, кандидат технических наук, доцент
(Краснодар, Российская Федерация)
e-mail: wadimhaikow@inbox.ru
ORCID iD: 0000-0003-1433-3562

DOI: XXXXXXXXXXX

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ВИД СТАТЬИ: научная статья
ЯЗЫК СТАТЬИ: русский

Резюме:

Для пистолетов М54, М57 (7.62×25 Токарев патрон) собраны баллистические параметры характеризующие их баллистику. В статье дан расчёт внешнебаллистических траекторий для двух законов сопротивления воздуха: 1943 года и Сиаччи, при этом использованы разные виды их математической записи (классические аналитические формулы, формулы кусочного вида, а также функции-таблицы). Для решения баллистической системы дифференциальных уравнений при табличном задании функции сопротивления воздуха используются сплайны.

Для закона 1943 года показана графическая интерпретация функции $C_d(i,v)$ в виде поверхности и её основные элементы. Показано, что такую поверхность можно построить для любого закона сопротивления воздуха.

Показан способ графического сравнения баллистических траекторных параметров. Все вычисления выполнены в среде

Mathcad 15, в статье приведён программный код расчёта. Показано, что за счёт подбора баллистических коэффициентов можно получить достаточно близкие по форме траектории. Однако в связи с тем, что движение пули по каждой из них определяется разными законами сопротивления воздуха, то замедление пули на каждой из них будет иметь свою собственную независимую форму и поэтому не совпадать с «траекторией-двойником».

Ключевые слова: *внешняя баллистика, пистолет M54, пистолет M57, патрон 7.62×25 TT, закон сопротивления воздуха, траектория пули, сплайн, Mathcad.*

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